Two- and Three-Body Correlations in Neutron Matter

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In the physics of neutron stars the density dependence of the energy density, $\varepsilon(\rho)$, (equation of state) plays a central role. The stiffness against compression is determined by the neutron-neutron interaction which is strongly repulsive at short distances. The theoretical treatment is involved because the repulsion induces short ranged two- and three-body correlations which cannot be understood in a mean-field picture.

Different many-body theories have been developed to treat these correlations, like Variational-Monte-Carlo (VMC) or Fermi-Hypernetted-Chain (FHNC) methods [1]. In 1973 H.A. Bethe proposed the homework problem to calculate the equation of state for neutron matter with the purely repulsive two-body interaction $V(r) = 9263.1\,\mathrm{MeV\,fm}\exp\{-4.9\,\mathrm{fm}^{-1}r\}/r$ in order to compare the different models.

The Unitary Correlation Operator Method (UCOM) [2] which has been quite successful in finite nuclei and Helium drops is used here for this interaction which has no attractive region and hence no bound state. Thus the form of the unitary correlator $C[\mathcal{R}_+(r)]$ cannot be found anymore by minimizing the energy of the two-body bound state with respect to the shape of $\mathcal{R}_+(r)$. For a binding potential the correlator shifts probability ($r \mapsto \mathcal{R}_+(r)$) out of the repulsive into the attractive region in order to obtain the minimum in the energy

$$E = \big\langle\Phi\big|\, C[\mathcal{R}_+(r)]^{-1}\,\big(T+V\big)\,C[\mathcal{R}_+(r)]\,\big|\Phi\big\rangle. \qquad (1)$$

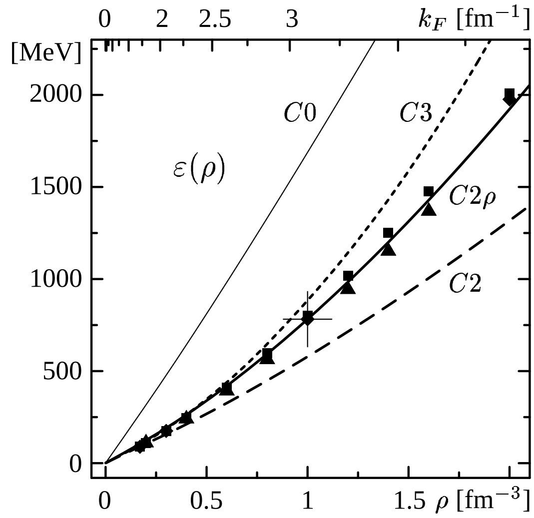
For Bethe's repulsive potential there is no minimum and in the lowest energy state ($E = 0$) the particles are simply shifted so far away from each other that they do not interact anymore. Therefore we minimize $\varepsilon = E/N$ of neutron matter where a real minimum exists because the average distance between the particles is fixed by the given density ρ.

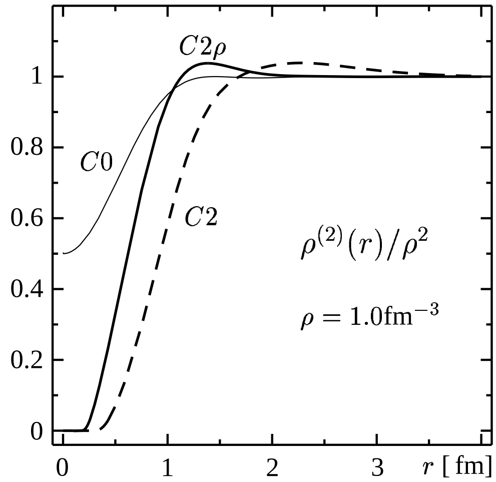
The upper figure compares our UCOM results with existing FHNC and VMC calculations (squares, triangles and diamonds). The uncorrelated mean-field energy $\varepsilon^{C0}(\rho)$ is of course much too repulsive. Including two-body correlations only ($C2$) does not give enough repulsion for densities $\rho > 0.5\,\mathrm{fm}^{-3} \approx 3\rho_0$ (nuclear matter: $\rho_0 = 0.16\,\mathrm{fm}^{-3}$). The inculsion of three-body correlations ($C3$) improves the comparison at low densities but overcompensates beyond $1\,\mathrm{fm}^{-3} \approx 6\rho_0$. However, a slightly density-dependent shift function $\mathcal{R}_+(r)$ which is used in an effective two-body correlation ($C2\rho$) can reproduce the numerically costly many-body calculations quite well over the whole density range.

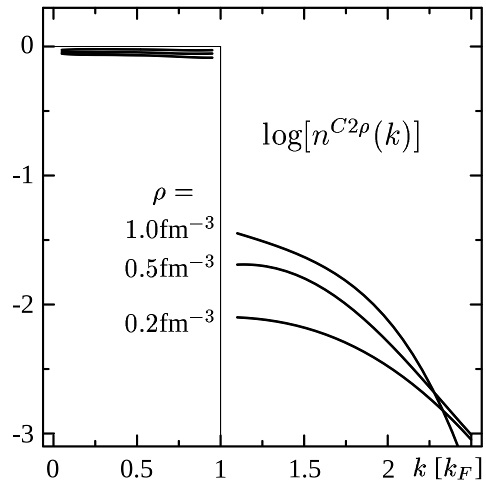
The middle figure shows the two-body density $\rho^{(2)}(r)$ as a function of the distance r between two neutrons. One sees that two-body correlations alone ($C2$) result in a rather large hole which is responsible for the lack of repulsion. But with a certain probability a third and fourth neutron is present in the correlation hole of the two neutrons. This effect is taken into account in the density-dependent correlator, which reproduces the energy density and thus includes effectively three- and more-body correlations ($C2\rho$).

The lower figure displays the single-particle occupation numbers of the momentum states as function of k for three different densities. Due to the short range repulsion momenta above the Fermi momentum k_F are occupied with increasing strength for higher densities.

The results show that the Unitary Correlation Operator Method reproduces the much more expensive FHNC and VMC calculations well enough so that more realistic interactions can be investigated.







[1] J.G. Zabolitzky; Phys. Rev. A 16 (1977) 1258
[2] H. Feldmeier, T. Neff, R. Roth and J. Schnack; Nucl. Phys. A632(1998)61