Structure of Degenerate Boson-Fermion Mixtures

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Recent experimental successes in the trapping and cooling of mixtures of bosonic and fermionic atoms [1,2] constitute a new branch in the field of trapped ultracold gases. Similar to the purely bosonic gases boson-fermion mixtures offer unique possibilities to study fundamental quantum phenomena. One of the ultimate experimental goals is the observation of a BCS transition of the fermionic component to a superfluid state, which may be achievable by sympathetic cooling of the fermionic component in a boson-fermion mixture.

Within the mean-field approximation using an effective contact interaction [3] the density distribution $n_{\rm F}(\vec{x})$ of the fermionic component is given by

$$n_{\rm F}(\vec{x}) = \frac{\sqrt{2m^3}}{3\pi^2}\left[\mu_{\rm F} - U_{\rm F}(\vec{x}) - \frac{4\pi a_{\rm BF}}{m} n_{\rm B}(\vec{x})\right]^{3/2}, \quad (1)$$

where $U_{\rm F}(\vec{x})$ is the external trapping potential and $\mu_{\rm F}$ is the chemical potential of the fermionic component. We have assumed equal mass m for both species. The interaction between the bosonic and fermionic atoms – characterized by the boson-fermion s-wave scattering length $a_{\rm BF}$ – couples the bosonic and fermionic density. The boson density $n_{\rm B}(\vec{x}) = \Phi_{\rm B}^2(\vec{x})$ is obtained from a modified Gross-Pitaevskii equation

$$\Big[-\frac{1}{2m}\vec{\nabla}^2 + U_{\rm B}(\vec{x}) + \frac{4\pi a_{\rm BF}}{m} n_{\rm F}(\vec{x}) + \frac{4\pi a_{\rm B}}{m}\Phi_{\rm B}^2(\vec{x})\Big]\Phi_{\rm B}(\vec{x}) = \mu_{\rm B}\Phi_{\rm B}(\vec{x}). \quad (2)$$

here $\mu_{\rm B}$ is the chemical potential of the bosons and $a_{\rm B}$ the s-wave boson-boson scattering length. We solve these coupled nonlinear differential equations through an efficient imaginary time evolution algorithm. In order to keep the discussion simple we restrict ourselves to spherical symmetric systems with identical parabolic trapping potentials $U_{\rm B}(x) = U_{\rm F}(x) = x^2/(2m\ell^4)$. The oscillator length $\ell = (m\omega)^{-1/2}$ serves as fundamental length unit for the numerical treatment.

First we consider the case of repulsive boson-boson and attractive boson-fermion interactions ($a_{\rm B} \geq 0, a_{\rm BF} < 0$). The upper row of Fig. 1 shows the density profiles of (meta)stable configurations with $N_{\rm B} = N_{\rm F} = 10^4$ particles for three different values of the boson-fermion scattering length $a_{\rm BF}$ and fixed $a_{\rm B}/\ell = 0.001$ (corresponds to $a_{\rm B} \approx 20\, a_{\rm Bohr}$ for a typical trap with $\ell = 1\,\mu$m). Due to the Pauli principle the fermionic density distribution is much more spread out and has a significantly lower central density than the bosonic distribution with the same particle number [notice the different scales for $n_{\rm B}(\vec{x})$ and $n_{\rm F}(\vec{x})$ in Fig. 1]. Attractive boson-fermion interactions generate an attractive mean-field for bosons proportional to the density of the fermions and vice versa. This causes an increase of both densities in the overlap region as can be seen in panel (b) of Fig. 1. With increasing strength of the boson-fermion attraction the fermion density grows substantially. As shown in Fig. 1(c) the fermion density can easily be increased by a factor 3 compared to the noninteracting case.

Second we consider mixtures with attractive boson-boson and repulsive boson-fermion interactions ($a_{\rm B} < 0$, $a_{\rm BF} \geq 0$). The ^{6}Li/^{7}Li mixture used in the experiment of Truscott *et al.* [1] belongs to this class of interactions. The lower row of Fig. 1 shows the density profiles for three different values of $a_{\rm BF} \geq 0$. Already for a very weak boson-fermion repulsion the two species separate spatially [see Fig. 1(e)], the bosons occupy the central region of the trap (boson core) and the fermions constitute a shell around it. The fermionic shell compresses the boson core, i.e. increases the maximum boson density as it is clearly seen in Fig. 1(f).

Any attractive interaction component — either boson-boson or boson-fermion interaction — can induce a mean-field instability of the degenerate mixture. In this case the attractive mean-field is not stabilized by the positive kinetic energy contribution any more, i.e. the gas can lower its energy by contracting and increasing the density in the central region. This collapse sets severe limitations on the numbers of bosons and fermions in order to retain the stability of the mixture. Thus it restricts that parameter ranges in which sympathetic cooling schemes can be applied experimentally. We investigate these phenomena in detail in Refs. [4,5].

[1] A.G. Truscott *et al.*; Science 291 (2001) 2570.

[2] F. Schreck *et al.*; Phys. Rev. Lett. 87 (2001) 080403.

[3] R. Roth, H. Feldmeier; Phys. Rev. A 64 (2001) 043603.

[4] R. Roth, H. Feldmeier; Phys. Rev. A 65 (2002) 021603(R).

[5] R. Roth; *in preparation.*

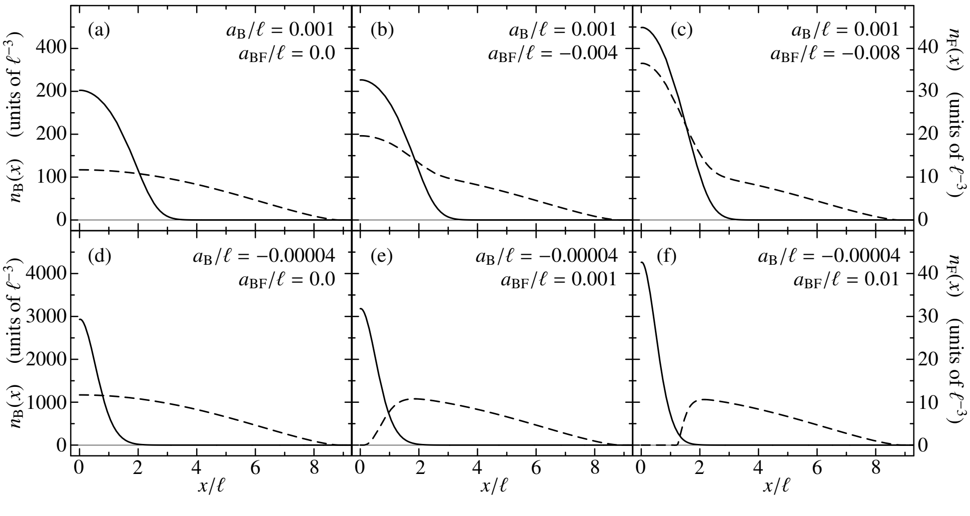

Fig. 1: Radial density profiles of a boson-fermion mixture with $N_{\rm F} = N_{\rm B} = 10\,000$ for different interaction strengths. The boson density $n_{\rm B}(\vec{x})$ is given by the solid line (left scale) and the fermion density $n_{\rm F}(\vec{x})$ by the dashed line (right scale). The upper row shows examples with increasing boson-fermion attraction and fixed $a_{\rm B}/\ell = 0.001$. The lower row depicts examples with increasing boson-fermion repulsion and $a_{\rm B}/\ell = -0.00004$.