D. Gazda · J. Mareš · P. Navrátil · R. Roth · R. Wirth

No-Core Shell Model for Nuclear Systems with Strangeness

Abstract We report on a novel *ab initio* approach for nuclear few- and many-body systems with strangeness. Recently, we developed a relevant no-core shell model (NCSM) technique [1] which we successfully applied in first calculations of lightest Λ hypernuclei. The use of a translationally invariant finite harmonic oscillator (HO) basis allows us to employ large model spaces, compared to traditional shell model calculations, and use realistic nucleon–nucleon (NN) and nucleon–hyperon (NY) interactions (such as those derived from EFT [2]). We discuss formal aspects of the methodology, show first demonstrative results for $^3_\Lambda$H, $^4_\Lambda$H and $^4_\Lambda$He, and give outlook.

Keywords hypernucleus · ab initio calculations · no-core shell model · chiral EFT

1 Introduction

One of the major goals of contemporary theoretical hypernuclear physics is to calculate the properties of hypernuclei starting from the bare interactions between nucleons and hyperons. However, the NY interaction is not well understood due to the very limited NY scattering database. Hypernuclei therefore provide an important source of constraints on and serve as a testing ground for the NY interaction, as well as the YY interaction where no scattering data are available. Chiral perturbation theory seems to be a promising bridge which allows us to connect QCD in the strangeness sector with low-energy hypernuclear physics [2; 3]. Moreover, lattice QCD simulations are approaching a level where they can provide additional constraints on the baryon–baryon interaction [4]. In the near future, a significant part of the research program in new facilities, like J-PARC in Japan and FAIR in Germany, will be devoted to strangeness physics. The proposed experiments will, in part, focus on accurate measurements of energy spectra of $S = -1$ and $S = -2$ hypernuclei [5; 6]. To properly connect the characteristics of hypernuclei with the underlying NY and YY interactions reliable calculations with realistic interactions are necessary. Nevertheless, only very few *ab initio* or *exact* calculations of hypernuclei are available for 3- and 4-body systems [7; 8; 9; 10; 11]. For $A \geq 4$ only a handful of *ab initio* approaches are applicable.

D. Gazda
ECT*, Villa Tambosi, I-38123 Villazzano (Trento), Italy
Nuclear Physics Institute, 25068 Řež, Czech Republic
E-mail: gazda@ujf.cas.cz

J. Mareš
Nuclear Physics Institute, 25068 Řež, Czech Republic

P. Navrátil
TRIUMF, 4004 Wesbrook Mall, Vancouver, BC V6T 2A3, Canada

R. Roth
Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

R. Wirth
Institut für Kernphysik, Technische Universität Darmstadt, 64289 Darmstadt, Germany

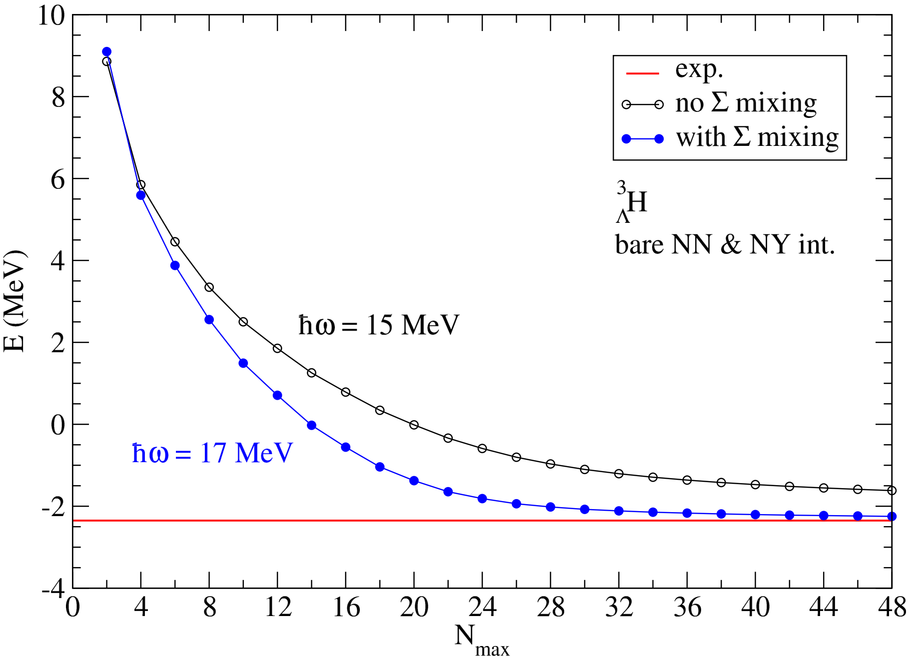

Fig. 1 (color online) The hypertriton ground state energy E as a function of the size of the model space $N_{\max}\hbar\omega$ calculated with the chiral NN and NY forces. Results of calculations in the presence (absence) of the explicit $\Lambda - \Sigma$ hyperon mixing in the hypernuclear state are denoted by full (open) circles for the optimal HO frequencies. The measured value of the hypertriton ground state energy is shown by straight line.

In Section 2, the methodology used in our calculations is briefly introduced, and in Section 3 we present first demonstrative results. Brief summary with outlook is given in Section 4.

2 Methodology

Our approach is based on the no-core shell model technique [1] which we have extended to incorporate strangeness degrees of freedom. The no-core shell model technique is one of the most powerful and universal *ab initio* methods in nuclear structure calculations. In the NCSM, the total Hamiltonian of the system is diagonalized in a *finite* A-body HO basis which is truncated by a maximal HO excitation energy $N_{\max}\hbar\Omega$ with respect to the unperturbed ground state of the A-body system. In the present calculations we utilize the relative Jacobi-coordinate HO basis, which enable us to perform calculations in larger model spaces compared to calculations in a conventional Slater-determinant HO basis since the center-of-mass degrees of freedom are explicitly removed and a basis coupled to a total angular momentum and isospin is used . Unlike standard shell model calculations, in the NCSM all particles are treated as active. This allows us to employ realistic NN and NY interactions. In this work we use the chiral N^{3}LO NN interaction [12] and the chiral LO NY interaction with cutoff $\Lambda = 600$ MeV [2]. In the following calculations the potentials are used in a “bare” form without constructing any effective interaction tailored to a specific model space. The results of calculations are thus variational with respect to the model space truncation.

3 Results and Discussion

Our aim is to develop an *ab initio* method for heavier s- and p-shell hypernuclei. In our initial calculations we focus primarily on the applicability of the NCSM to nuclear systems with nonzero strangeness, particularly on the convergence properties and their dependence on the size of the model space. We performed calculations of hypertriton $^3_\Lambda$H, $^4_\Lambda$H and $^4_\Lambda$He with realistic chiral NN and NY potentials.

In Fig. 1, the hypertriton ground state energy is shown as a function of the size of the model space, calculated using the chiral NN and NY interactions. The results of calculations with the

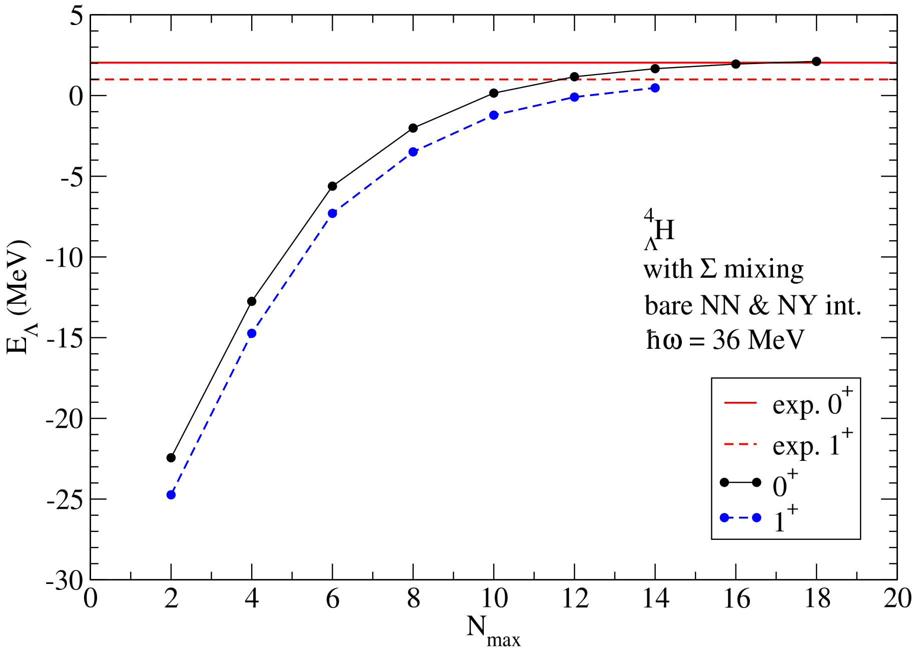

Fig. 2 (color online) The Λ separation energy E_Λ in the 0^+ (full line) and 1^+ (dashed line) states of ${}^4_\Lambda$H as a function of the size of the model space $N_{\max}\hbar\omega$, for the optimal HO frequency, calculated with the chiral NN and NY forces. The measured values of the corresponding Λ hyperon separation energies are shown by straight lines.

explicit Σ hyperon mixing in the hypertriton state (full circles) are compared to those calculated in the absence of explicit $\Lambda-\Sigma$ mixing (open circles). The inclusion of the $\Lambda-\Sigma$ hyperon mixing is essential in light hypernuclear systems [14]. Due to the extremely weak binding of the hypertriton, $E^{(\mathrm{exp.})}({}^3_\Lambda\mathrm{H}) = -2.35 \pm 0.05$ MeV [13], the convergence with $N_{\max}$ is very slow, as illustrated in the figure. Preliminary extrapolation of the hypertriton ground state energy to the infinite model space limit gives $E({}^3_\Lambda\mathrm{H}) = -2.31 \pm 0.01$ MeV.

Fortunately, in case of heavier and more bound and compact systems, such as ${}^4_\Lambda$H and ${}^4_\Lambda$He, the convergence with the size of the model space is much faster, as shown in Figures 2 and 3. Here the Λ hyperon separation energies, $E_\Lambda = E({}^{A-1}Z) - E({}^A_\Lambda Z)$, in the 0^+ and 1^+ states of ${}^4_\Lambda$H and ${}^4_\Lambda$He are shown as functions of the size of the model space, calculated with the chiral NN and NY interactions. The results of calculations approach the measured Λ hyperon separation energies in the $0^+(1^+)$ state of ${}^4_\Lambda$H and ${}^4_\Lambda$He, $E_\Lambda^{(\mathrm{exp.})}({}^4_\Lambda\mathrm{H}) = 2.04\pm0.04\,(1.00\pm0.06)$ MeV and $E_\Lambda^{(\mathrm{exp.})}({}^5_\Lambda\mathrm{He}) = 2.39\pm0.03\,(1.24\pm0.05)$ MeV [13].

The results of our calculations demonstrate that the NCSM formalism presents a powerful and promising tool to study nuclear systems with nonzero strangeness. It is rather straightforward, and currently under development, to extend the NCSM methodology to heavier, $A \geq 4$, hypernuclei. Nevertheless, more work is needed to carry out systematic studies of the underlying interactions, perform calculations in larger model spaces or employ effective interactions, and include (possibly) NNY forces. Full details of the methodology and extrapolation of results to the infinite model space limit will be discussed elsewhere.

4 Summary

The no-core shell model technique for nuclear systems with nonzero strangeness was developed and applied in calculations of very light Λ hypernuclei. Our first results for ${}^3_\Lambda$H, ${}^4_\Lambda$H and ${}^4_\Lambda$He using chiral NN and NY forces look promising and give reasonable description of experimental data. The NCSM approach seems to be a viable candidate for the *ab initio* description of hypernuclei in the $A \geq 4$ mass region. In case of ${}^3_\Lambda$H the convergence with the size of the model space is slow as expected from the weak

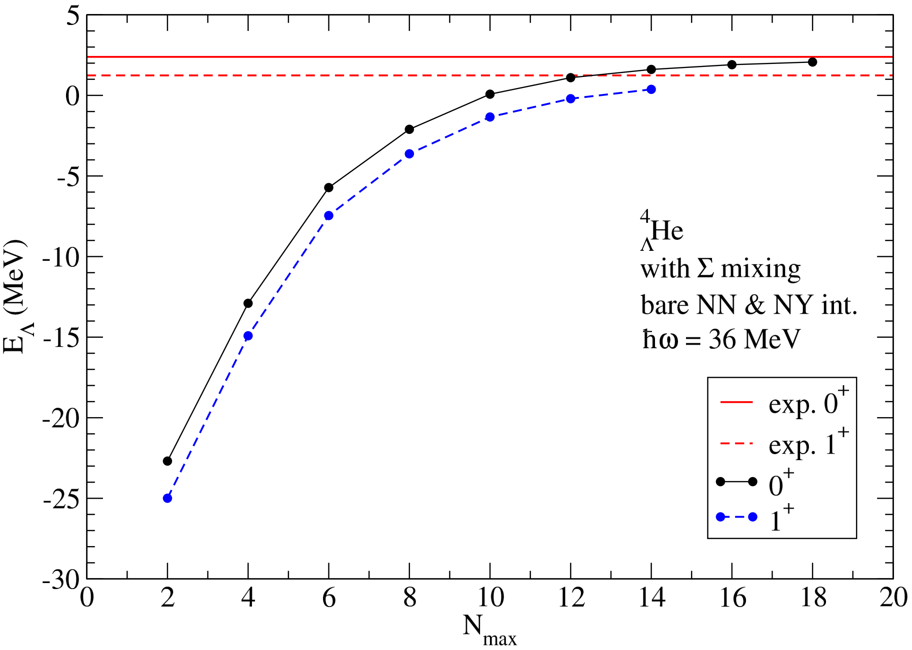

Fig. 3 (color online) The Λ hyperon separation energy E_Λ in the 0^+ (full line) and 1^+ (dashed line) states of $^{4}_{\Lambda}$He as a function of the size of the model space $N_{\rm max}\hbar\omega$, for the optimal HO frequency, calculated with the chiral NN and NY forces. The measured values of the corresponding Λ hyperon separation energies are shown by straight lines.

binding of the system. The convergence is much faster for the heavier and more bound hypernuclei $^{4}_{\Lambda}$H and $^{4}_{\Lambda}$He.

In future we intend to extend our calculations to heavier s- and p-shell hypernuclei and possibly study also other 'exotic' systems, such as $\bar{K}$- and η-nuclei.

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